NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

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AN ANALYSIS OF THE COMPRESSIVE STRENGTH OF HONEYCOMB CORES FOR SANDWICH CONSTRUCTIONS

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SUMMARY

The analysis described herein was undertaken as a part of an investigation of low-density fibrous core materials for sandwich construction. In this investigation it was found that honeycomb structures exhibit promise as core materials because of their high strength-weight ratios. A formula has been derived that reduced considerably the amount of testing necessary to evaluate the effectiveness of various fibrous materials of which the honeycomb cores were made. This formula relates compressive strengths of honeycomb cores having the same cell shape but having various combinations of size of cell and thickness of cell wall.

This analysis is partly empirical, being based upon data obtained from tests of plywood panels. It was applied successfully to resin-impregnated papers, but should be verified for materials greatly different from these before it is applied generally.

INTRODUCTION

The honeycomb-type structures used in this investigation were made by bonding together sheets of corrugated resin-impregnated paper as indicated in figure 1, with the walls of the cells made of the resin-impregnated paper, the resins cementing succeeding sheets. In use, the axes of the cells are perpendicular to the faces of the sandwich. The honeycomb core must have sufficient compressive and tensile strength in the direction of the axes to hold the facing materials at the desired separation and prevent them from wrinkling when subject to compression or shear acting in planes perpendicular to the axes of the cells.

In such a honeycomb construction the shape and size of the cells and the thickness of the cell walls can be varied. A change in any one of these may be expected to change the strength of the honeycomb. A formula has been derived that relates compressive strengths among such honeycomb cores having the same cell shape but having various combinations of size of cell and thickness of cell wall.

Availability and experimental verification of this formula make possible a reduction of the testing required to establish values of compressive strengths for various honeycomb constructions, inasmuch as tests of one combination of variables make it possible to compute the strength of any construction in which the cell shape and the material of the cell walls are the same.

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THEORETICAL DERIVATION OF FORMULA

The development of the formula is as follows:

It is assumed that under compression each cell wall will act independently like a plate supported along its edges and loaded at its ends and that the compressive strength of the honeycomb will be determined by the failing stress of the plate.

The critical buckling stress of such a plate is expressed by the following formula (reference 1)

$$p_{cr} = KE\frac{h^2}{a^2} \tag{1}$$

where

p_{cr} critical stress of plate

a width of plate

h thickness of plate

E modulus of elasticity of the material

K a constant depending upon the type of edge support of the plate

For the formula to be generally applicable to honeycomb constructions, the value of K must take into account the narrow walls of double thickness at the junctions of corrugations, the wider walls of single thickness, and the fact that the wider walls may be curved rather than flat.

In general, experience and observation show that the failing stress exceeds the critical buckling stress. To get an estimate of this excess, use is made of data from tests of plywood as given in table 28 of reference 2. These data are plotted logarithmically in figures 2 and 3 which relate to loadings parallel and perpendicular, respectively, to the face grain of the plywood. In each figure the abscissa is the observed critical stress and the ordinate is the average stress (across the width of the plate) at failure, each expressed as a ratio to the compressive strength of the plywood. It is evident that the plotted points, which cover a wide range on the abscissa scale, can be represented with reasonable accuracy by a straight line. An additional line was drawn through the point 1,1 at a slope of 1 to 1 in each graph. Ordinates to the lines with a slope of 1 to 1 represent the stress at which buckling will occur, since these lines pass through points for which the abscissas equal the ordinates. The same data are shown upon rectangular coordinates in figures 4 and 5.

It may be noted that the two lines in each figure cross at about the proportional limit of the material (about two-thirds of the compressive strength) and that the slopes of the curves of stress at failure are each about one-third. A good approximation of the data is given by the equation,

$$\frac{p}{p_u} = \left(\frac{p_p}{p_u}\right)^{2/3}\left(\frac{p_{cr}}{p_u}\right)^{1/3} \tag{2}$$

or

$$p = p_p^{\ 2/3}\ p_{cr}^{\ 1/3} \tag{3}$$

where

p average stress at failure

p_u compressive strength of material

p_p proportional limit of material

Equation (3) may give values of p greater than p_u, and it is not valid in the range in which it does. It is valid only if the computed critical stress is less than the proportional limit of the material, as is shown by the plotted points in figures 2 to 5.

By substitution of equation (1) in (3),

$$p = p_p^{2/3} (KE)^{1/3} \left(\frac{h}{a}\right)^{2/3} \tag{4}$$

and the specific compressive strength is

$$p_s = \frac{p}{g} = \frac{p_p^{2/3} (KE)^{1/3}}{g} \left(\frac{h}{a}\right)^{2/3} \tag{5}$$

where g is the specific gravity of the impregnated paper, and p_s may also be considered the specific compressive strength of a honeycomb construction, since

$$\frac{p_a}{g_a} = \frac{p}{g}$$

where p_a is the apparent compressive strength in pounds per square inch and g_a is the apparent specific gravity of the core construction.

Equation (5) contains the width of the plate a and the thickness of the cell wall h. Since the plates in practical honeycomb cores are not flat, it is impossible to determine the proper widths of the individual plates. This width, however, can be considered proportional to any cross-sectional dimension of the cell. The proportionality factor will be different for cells of different shape, but will not change with cell size or wall thickness. For purposes of convenience in connection with cores made of corrugated sheets cemented together, the plate width a will be considered as proportional to the height of the corrugation α, that is,

$$a = n\alpha \tag{6}$$

It is difficult to measure accurately the thickness of the cell walls in a honeycomb core. This thickness, however, can be expressed in terms of the apparent specific gravity of the core and the specific gravity of the material from which the core is made. The apparent specific gravity can be calculated from the weight and gross dimensions of a piece of the core, and the specific gravity of the material can be calculated from the weights of a piece of the core in air and submersed in a liquid.

Figure 6 is a sketch of a section of a half cell, or one complete corrugation of the corrugated material used in the manufacture of the core. The weight of this section is

$$w = ru\alpha hbgq$$

where q is the density of water and r, u, α, h, and b are as shown in the figure.

The gross volume of the piece shown in figure 6 is

$$v = (\alpha + h)\ u\alpha b$$

and the apparent specific gravity is

$$g_a = \frac{w}{vq} = \frac{rhg}{\alpha + h} \qquad (7)$$

Then

$$\frac{h}{\alpha} = \frac{g_a}{rg - g_a} \qquad (8)$$

in which r is the ratio of the developed (original) length of the corrugated sheet to the length of the sheet after corrugation. This ratio can be determined in a number of ways.

By using equations (6) and (8), equation (5) becomes

$$p_s = \frac{(KE)^{1/3}}{g}\left(\frac{p_p}{n}\right)^{2/3}\left(\frac{g_a}{rg - g_a}\right)^{2/3} \qquad (9)$$

The values of n, K, and r are related to the shape of the cells. The first two can be combined into a single constant that can be determined experimentally. Formula (9) for the specific crushing strength of the honeycomb core then becomes

$$p_s = \frac{s}{g}E^{1/3}\left[\frac{p_p\ g_a}{rg - g_a}\right]^{2/3} \qquad (10)$$

in which

$$s = \left(\frac{K}{n^2}\right)^{1/3}$$

This formula can be simplified further if the honeycomb cores are all made of the same material: thus,

$$p_s = c\left[\frac{g_a}{rg - g_a}\right]^{2/3} \qquad (11)$$

in which $c = \frac{s}{g} E^{1/3} p_p^{2/3}$, and can be evaluated from experiments in which the other quantities in equation (11) have been measured.

The value of c will remain constant even for different materials, provided the modulus of elasticity and the proportional limit vary directly with the specific gravity. This is approximately true for a paper impregnated with a resin. If the resin content varies over a limited range, the modulus of elasticity and the proportional limit will be roughly proportional to the specific gravity.

The value of c is particularly useful in the comparison of two honeycomb cores having different cell shapes and made of different materials. If specimens of two such cores do not have identical apparent specific gravities, a comparison of their specific compressive strengths is not proper because the apparent compressive strength does not vary directly with the apparent specific gravity. A comparison of the values of c for the two cores, however, is accurate inasmuch as such a comparison yields a ratio identical to that which would be obtained if specimens of like specific gravities were compared. The dimensions of c are those of a specific stress and therefore c might be called the fundamental specific compressive stress.

EXPERIMENTAL VERIFICATION

Seven blocks of honeycomb core material were fabricated by using a polystyrene contact resin. Three of the blocks were made of chestnut chip paper and four of kraft paper. A different thickness of paper was used in each block to obtain core materials having different values of apparent specific gravity. The chestnut chip core materials were made in two different cell sizes of similar shape to obtain a greater range in apparent specific gravity than could otherwise be obtained. Specimens were cut from these blocks and tested in compression. The resulting data are given in table I.

From formula (11):

$$c = p_s \left[\frac{rg - g_a}{g_a}\right]^{2/3} \tag{12}$$

When appropriate values from table 1 are substituted in this formula,

$$c_1 = 5280 \left[\frac{1.23 \times 1.41 - 0.101}{0.101}\right]^{2/3} = 33{,}800$$

Similarly,

$$c_2 = 33{,}700$$

$$c_3 = 31{,}900$$

$$\text{Average} = 33{,}100$$

$$c_{F-3} = 26{,}300$$

$$c_{F-4} = 25{,}000$$

$$c_{F-5} = 23{,}100$$

$$c_{F-6} = 25{,}100$$

$$\text{Average} = 24{,}900$$

The reasonably close check of the values obtained for the constant c for the three cores made of chestnut chip paper and for the four cores made of kraft paper indicates that equation (11) is a reasonable one.

The higher values of c obtained from the core material in which the chestnut chip paper was used indicate that this paper is the better of the two. The average value of c for the core materials can be substituted in equation (11). For example, when the average value of c for the cores made of chestnut chip paper is substituted,

$$p_s = 33{,}100 \left[\frac{g_a}{1.23g - g_a}\right]^{2/3} \tag{13}$$

This equation will probably be valid for all honeycomb cores made of this material and having cells similar in shape to those of the cores tested.

Formula (13) can be used to determine the apparent specific gravity of this type of honeycomb core that will just meet a requirement of a specific compressive strength of 1000 psi. It may be assumed that the specific gravity of the paper will be the average of the values obtained from test specimens, or 1.422 for the chestnut chip paper. By substitution in formula (13),

$$1000 = 33{,}100 \left[\frac{g_a}{1.23 \times 1.422 - g_a}\right]^{2/3}$$

The apparent specific gravity is then found to be

$$g_a = 0.00916$$

Although this material would meet the specific compressive strength requirement, the apparent compressive strength would be only 9.16 psi, which is probably too low to be useful. Further, equation (8) yields in this case

$$\frac{h}{\alpha} = \frac{0.00916}{1.23 \times 1.422 - 0.00916} = 0.0053$$

Thus, the thickness of the cell wall would be about five-thousandths of the corrugation height. Such a honeycomb core is difficult to manufacture by methods available at the present time, and also, it is not certain that equation (13) applies accurately to honeycomb cores having such thin cell walls. Honeycomb cores of this type with thicker cell walls will have a specific compressive strength in excess of 1000 psi.

By using the average values of c, r, and g in formula (11) for each of the two types of honeycombs (chestnut chip and kraft paper base), values of apparent compressive strength for various apparent specific gravities were obtained, and the curves of apparent specific gravity against apparent compressive strength were drawn (fig. 7). Test values of the apparent compressive strength for the apparent specific gravities of the sample honeycombs are also shown. It may be noted that the experimental values deviate slightly from the curves. This deviation may be due in part to the variation in the specific gravity of the impregnated papers (col. 8, table I). In general, the data verify formula (11) reasonably well for honeycomb structures made of the same paper and contact resin and having cells of similar shape.

Forest Products Laboratory, Forest Service,
U. S. Department of Agriculture,
Madison, Wis., August 27, 1946.

REFERENCES

1. March, H. W.: Buckling of Flat Plywood Plates in Compression, Shear, or Combined Compression and Shear. Forest Prod. Lab. Rep. Mimeo. 1316, April 1942.

2. Norris, C. B., Voss, A. W., and McKinnon, P. F.: Supplement to Buckling of Flat Plywood Plates in Compression, Shear, or Combined Compression and Shear. Forest Prod. Lab. Rep. Mimeo. 1316-I, March 1945.

TABLE I.– SOME PROPERTIES AND DIMENSIONAL CHARACTERISTICS OF HONEYCOMB STRUCTURES FABRICATED FROM CHESTNUT CHIP AND KRAFT PAPERS WITH A POLYSTYRENE CONTACT RESIN

Base paper	Core	Number of test values	Corrugation flute type	Apparent compressive strength, p_a (psi)	Specific compressive strength, p_s (psi)	Apparent specific gravity of core, g_a	Specific gravity of material, g	Thickness of material (approx.), h (in.)	Height of corrugations plus thickness of material, $\alpha + h$ (in.)	Ratio of corrugation height to wave length, $\frac{l}{u}$	Ratio,[1] r
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Chestnut chip	1	2	A	533	5280	0.101	1.41	0.011	0.158	0.34	1.23
Do.	2	4	A	302	4080	0.074	1.49	0.007	0.158	0.36	1.23
Do.	3	2	B	1065	6920	0.154	1.37	0.008	0.085	0.33	1.23
Kraft	F-3	5	B	735	5120	0.143	1.41	0.004	0.082	0.294	1.28
Do.	F-4	5	B	779	5160	0.151	1.36	0.006	0.088	0.327	1.30
Do.	F-5	5	B	982	5410	0.181	1.33	0.008	0.097	0.372	1.34
Do.	F-6	5	B	1293	6100	0.212	1.35	0.009	0.098	0.386	1.47

[1]Ratio of the developed (original) length of the corrugated sheet to the length of the sheet after corrugation.

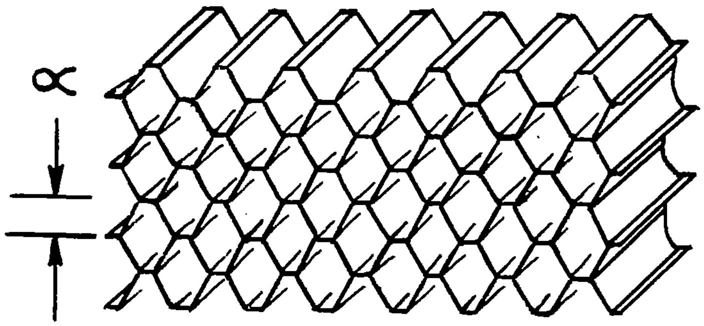

Figure 1.--Sketch of honeycomb core material.

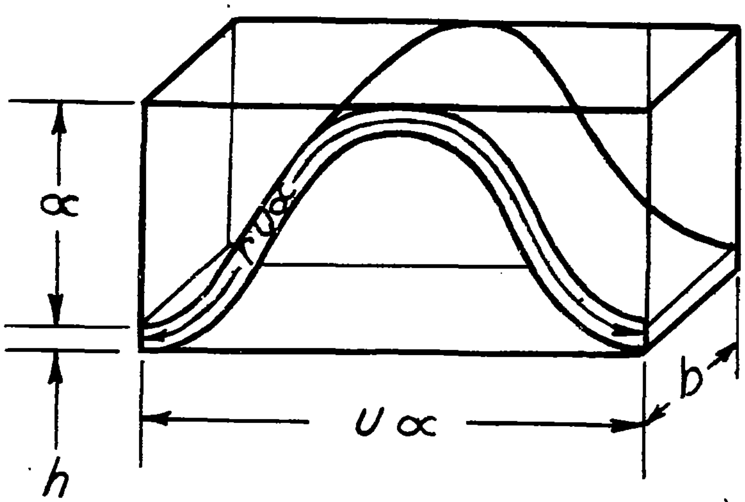

Figure 6.--Sketch of an element of honeycomb core material.

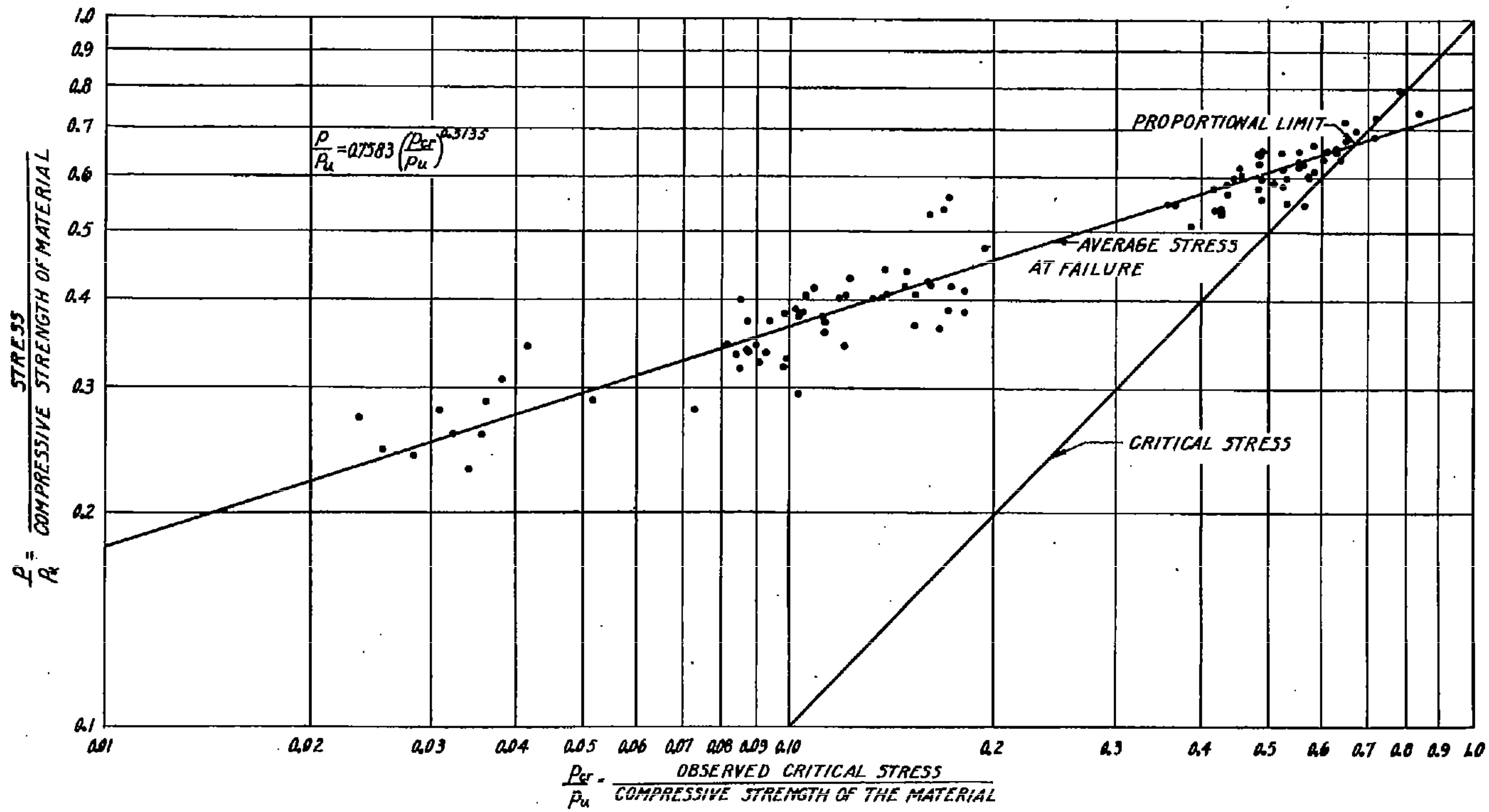

Figure 2.--Plywood plates in compression. Compressive stress parallel to face grain.

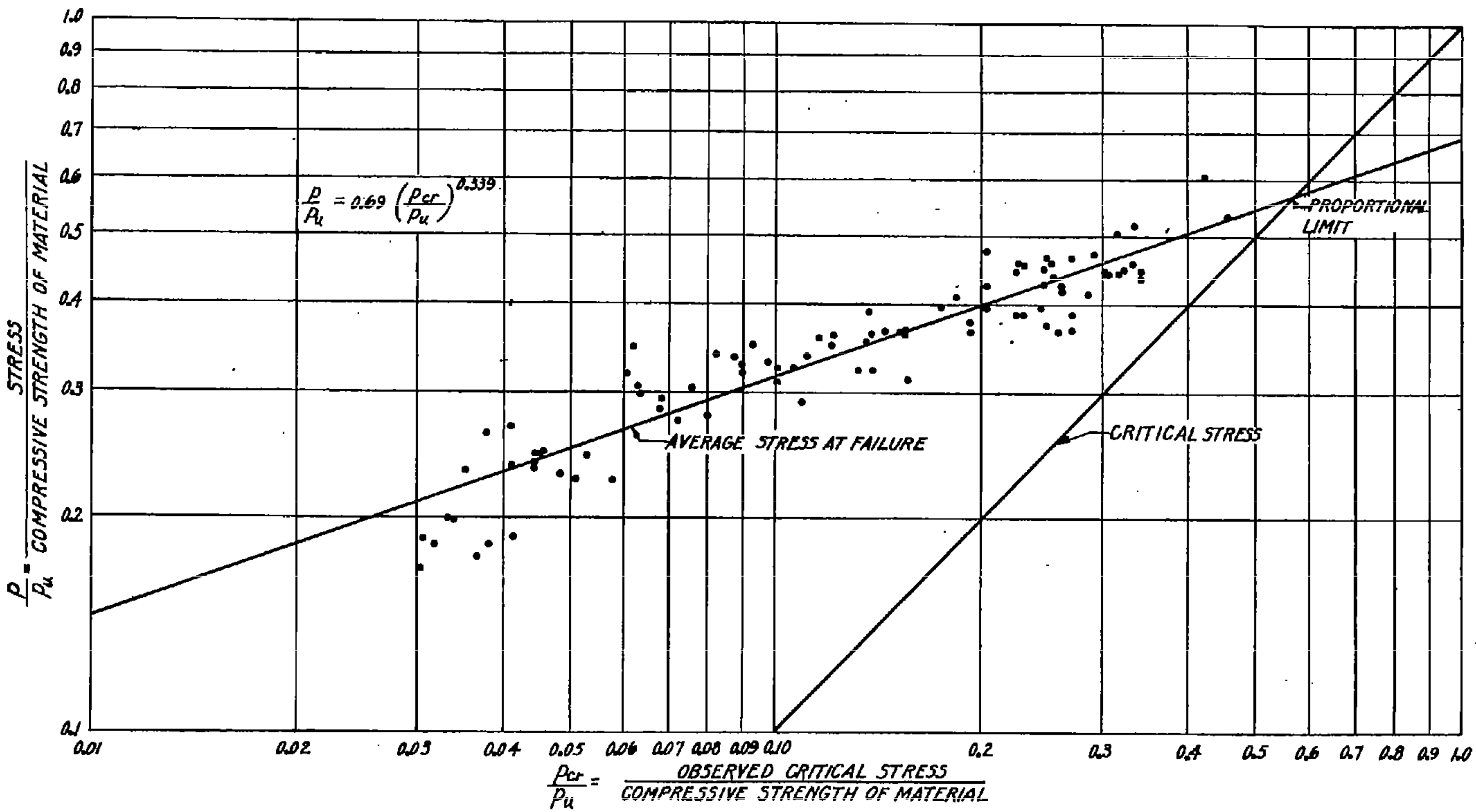

Figure 3.--Plywood plates in compression. Compressive stress perpendicular to face grain.

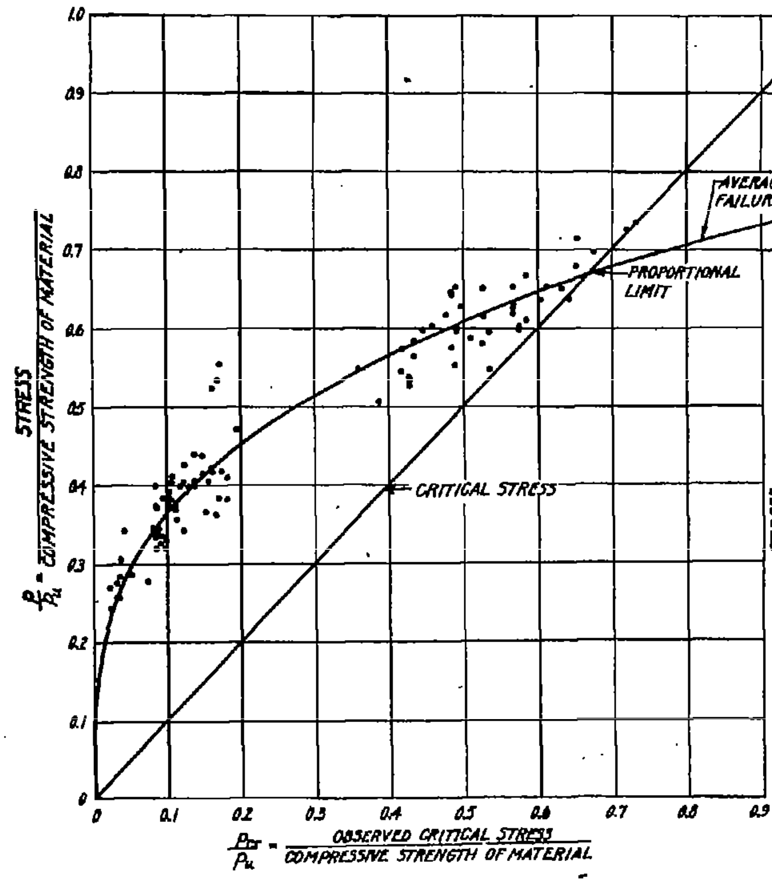

Figure 4.-Plywood plates in compression. Compressive stress parallel to face grain. (Same data as in fig. 2.)

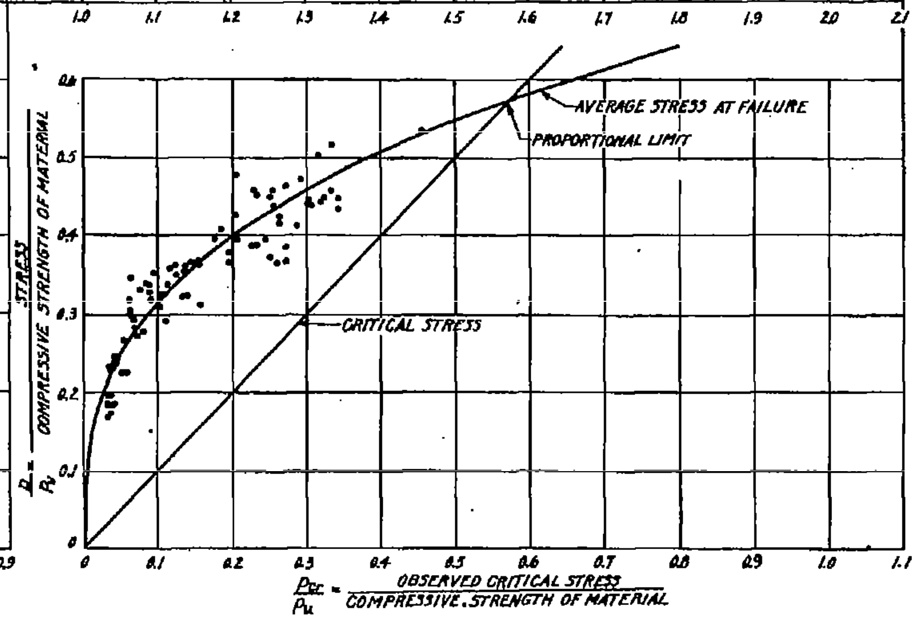

Figure 5.-Plywood plates in compression. Compressive stress perpendicular to face grain. (Same data as in fig. 3.)

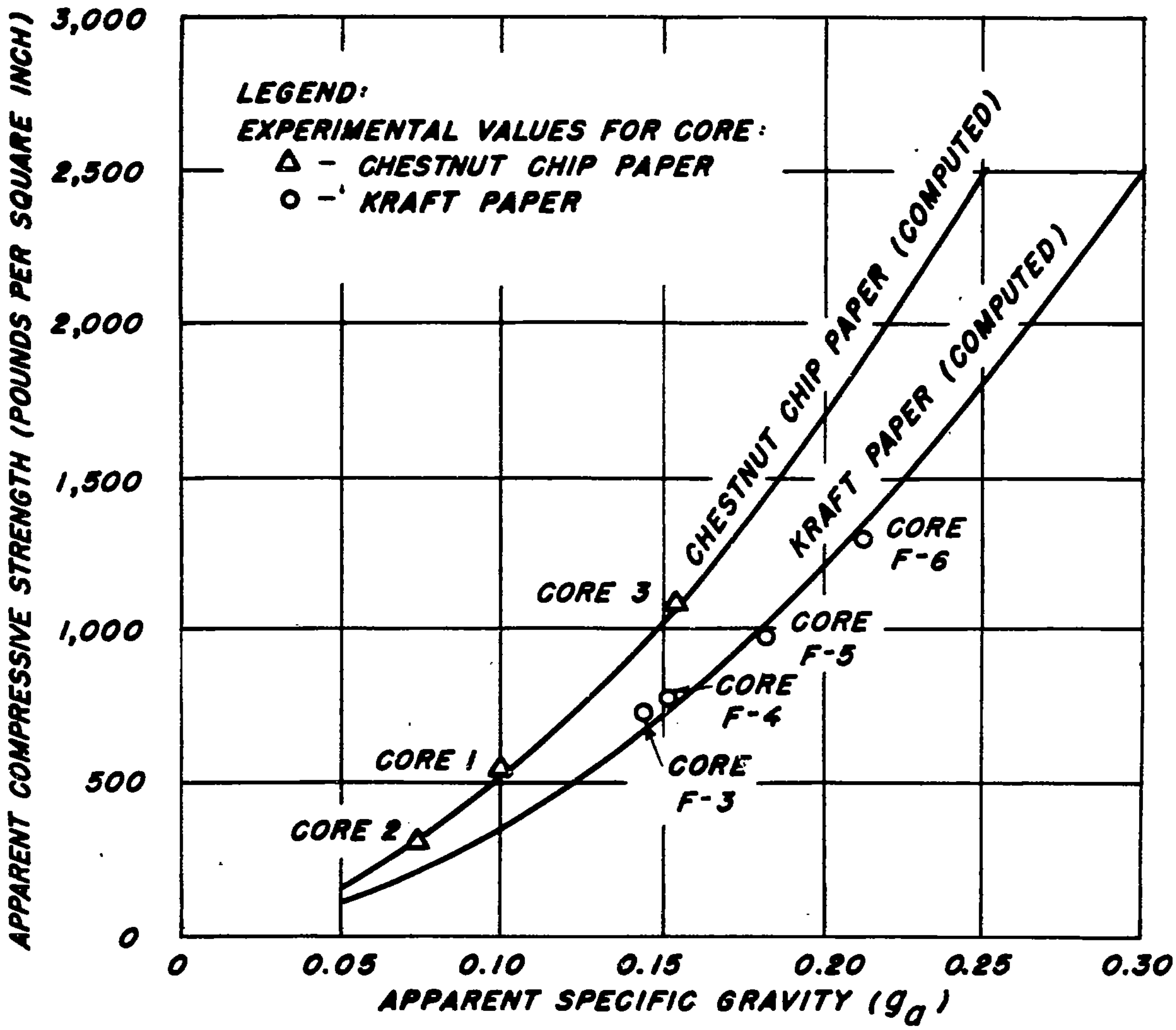

Figure 7.--Apparent compressive strength of honeycomb cores parallel to cell openings, plotted against apparent specific gravity. (Curves computed from formula (11).)